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Topologized Bipartite Graph

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Let G be bipartite graph with V vertices and E edges. A topological space X is a topologized graph if it satisfies the following conditions:

- Every singleton is open or closed.
- $\forall x \in X, |\partial(x)| \leq 2$, where $\partial(x)$ is a boundary of x.

Here the topology is defined on the graph, since the space X is the union of vertices and edges. This work is extended from topologized graph to star graph (0<n≤2), Bistar, Bipartite graph, Tree, complete bipartite graph.

Keywords: Topology; labeling; star; bipartite graph; complete bipartite graph.

AMS classification code: 97K30, 05C78.

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1 Introduction

In this paper we discussed about topologized graphs referred to Antoine Vella [1]. In 2005 Antoine Vella [1] tried to express combinatorial concepts in topological language. As a part the investigation classical topology, pre path, path [2], pre cycle, compact space, cycle space and bond space [3], locally connectedness and ferns were defined. Given a hypergraph H [4], the classical topology on$V_H \cup E_H$ is the collection of all sets U such that, if U contains a vertex v, then it also contains all hyperedges incident with v. It is interesting to note that all these topologies are either defined on the vertex set, or on the union of vertex set and edge set. For Topological graph theory, more important application can be found in printing electronic circuits where the aim is to print (embed) a circuit on a circuit board without two connections crossing each other and resulting in a short circuit. And the Bipartite graphs are extensively used in modern coding theory, especially to decode codewords received from the channel. Factor graphs and Tanner graphs are examples of this. A Tanner graph is a bipartite graph in which the vertices on one side of the bipartition represent digits of a codeword, and the vertices on the other side represent combinations of digits that are expected to sum to zero in a codeword without errors [5]. In this paper we have applied and verified some new methods of topology for bipartite graphs.

All graphs are finite, simple, undirected graphs with no loops and multiple edges and planar. Every topological space considered here are finite. And the topology is defined on the set X which is the union of vertices (V) and edges (E) of the graph G.

2 Preliminaries

2.1 Definition: [6]

A topology on a set X is a collection τ of a subset of X with the following properties:

i. Φ and X are in τ.
ii. The union of the element of any sub collection of τ is in τ (arbitrary union).
iii. The intersection of the element of any finite sub collection of τ is in τ.

The set X for which a topology τ has been specified is called a topological space.

2.2 Definition: [1]

A topologized graph is a topological space X such that

- Every singleton is open or closed.
- $\forall\ x \in X$, $|\partial(x)| \leq 2$. since $\partial(x)$ is denoted by the boundary of a point x

2.3 Definition: [4]

A hyperedge of a topological space is a point which open but not closed. A hyperedge of a topological space is an edge if its boundary consists of at most two points. An edge of a topological space is a loop if it has precisely one boundary point, proper edge otherwise.

2.4 Definition: [7]

A graph G is called a bigraph or bipartite graph if V can be partitioned into two disjoint subsets V and W such that every line of G joins a point of V to a point W. (V, W) is called a bipartition of G.

2.5 Theorem: [8,9]:

If G is a bipartite graph if and only if it contains only even cycle.

2.6 Proposition: [10]

Every tree is a bipartite graph

2.7 Definition: [11]

A graph G is called complete bipartite if G contains every line joining the points of V to the point of W. If V contains m points and W contains n points then the complete bigraph G is denoted by $K_{m,n}$.

2.8 Definition: [7,12]

The graph $K_{1,n}$ is a star graph. It is the graph with n+1 vertices –a vertex of degree n is called the apex and n pendant vertices.

2.9: Definition:

Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$

3 Main Results

Theorem 3.1: $K_{1,n}$ is a topologized graph (for $0<n\leq 2$)

Proof: Let $K_{1,n}$ be a star graph. Let (X, τ) be a topological space with the topology defined by $V \cup E$. $\forall V$, $W \in V$. Let V be the centre of $K_{1,n}$ and $w_1, w_2, \ldots, w_n$ as its pendent vertices. V can be bipartite into every vertices of $w_1, w_2, \ldots, w_n$.

Let n=1 for $|X|=3$ since two vertices and one edge. Clearly for all $x \in X$, $|\partial(x)|\leq 2$ and every $x \in X$ is closed (or) open. Since {x} is closed then any point $y \in X \setminus \{x\}$ is open. The set E of points whichare not closed is open, and its complement V is closed.

Therefore $K_{1,1}$ is a topologized graph.

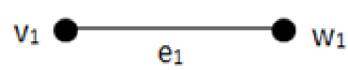
Let n=2 for $|X|=5$ since three vertices and two edges. Every singleton is open (or) closed. The boundary of all vertices and edges are at most two.

Therefore $K_{1,2}$ is a topologized graph.

Hence $K_{1,n}$ is a topologized graph. (for $0<n\leq 2$)

Example 3.1.1

Let $K_{1,1}$ be a star graph.



$K_{1,1}$ Topologized graph

$\forall$V, W$\in$V.V= {v_1}; W= {w_1}; E= {e_1};

Let v_1, w_1 denote the vertices, and e_1 the edges labeled so that $f_G(e_1) = \{v_1, w_1\}$

Let X= {v_1, e_1, w_1} be a topological space with the topology defined on

$$\tau = \{x, \phi, \{v_1\}\ \{w_1\}\ \{v_1, w_1\}\ \{v_1, e_1\}\}$$

For every $\{x\} \in X$is open or closed. Here the singleton vertices of {v_1} {w_1} is closed.

$\forall x \in X$, $|\partial(x)| \leq 2$
$\partial(v_1) = \{w_1\}$ so that, we have $|\partial(v_1)| = 1$;
$\partial(e_1) = \{v_1, w_1\}$ so that, we have $|\partial(e_1)| = 2$;
$\partial(w_1) = \{w_1\}$ so that, we have $|\partial(w_1)| = 1$;

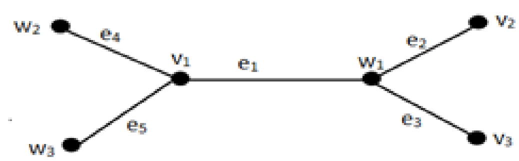
The boundary of every vertices and edges is at most two.

$K_{1,1}$ is topologized graph.

Theorem: 3.2: Bistar is not a topologized graph.

Proof: Let $B_{(n,n)}$ be a Bistar. Let (X, τ) be a topological space with the topology defined by $V \cup E$. Every singleton is open or closed. But the boundary of vertices is greater than two. Hence it is not topologized graph.

Example 3.2.1: Let $B_{(2,2)}$ is a bistar graph



$B_{(2,2)}$ non topologized

$\forall$V, W$\in$V. V= {v_1,v_2,v_3} ; W= {w_1,w_2,w_3}; E= {e_1,e_2,e_3,e_4,e_5}

Let v_1, v_2, v_3, w_1, w_2, w_3 denote the vertices and e_1, e_2, e_3,e_4, e_5 the edges labeled so that

$f_G(e_1) = \{v_1, w_1\}$; $f_G(e_2) = \{w_1, v_2\}$; $f_G(e_3) = \{w_1, v_3\}$; $f_G(e_4) = \{v_1, w_2\}$; $f_G(e_5) = \{v_1, w_3\}$;

Let X = {v_1, v_2, v_3, w_1, w_2, w_3, e_1, e_2, e_3, e_4, e_5} be a topological space with the topology defined on τ = {X, ϕ, {e_1},{e_2},{e_1,e_2},{v_1,e_1,w_1,e_2,v_2,e_3,v_3}

{w_3,e_5,v_1,e_1,w_2,e_4},{v_1,e_1},{,v_1,e_1,w_2,e_2,e_4,e_5,w_3},{v_1,e_1,e_2}

Here for every $\{x\} \in X$is open or closed.

$\forall x \in X$, $|\partial(x)| \leq 2$
$|\partial(v_1)| = \{w_1, w_3, w_2\} = 3$; $|\partial(v_2)| = \{w_1\} = 1$;
$|\partial(v_3)| = \{w_1\} = 1$; $|\partial(w_1)| = \{v_1, v_2, v_3\} = 3$;

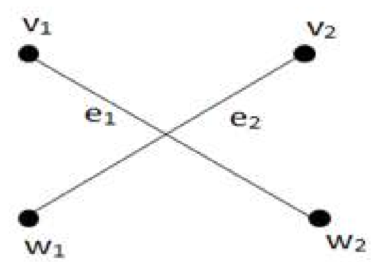
$|\partial(w_2)| = \{v_1\} = 1$; $|\partial(w_3)| = \{v_1\} = 1$;
$|\partial(e_1)| = \{v_1, w_1\} = 2$; $|\partial(e_2)| = \{w_1, v_2\} = 2$;
$|\partial(e_3)| = \{w_1, v_3\} = 2$; $|\partial(e_4)| = \{v_1, w_2\} = 2$;
$|\partial(e_5)| = \{v_1, w_3\} = 2$;

The boundary of vertices $\{v_1\}$ and $\{w_1\}$ is three. Therefore $B_{(2,2)}$ is not satisfied the boundary condition. Hence $B_{(2,2)}$ is not topologized.

Theorem: 3.4: Every bipartite graph is topologized if degree of every vertex is at most two.

Proof: Let G be a bipartite graph. Let (X, τ) be a topological space topology defined by $V \cup E$. G is a bipartite graph in which each vertex has degree at most two. Therefore, the boundary of every vertices and edges must be less than three. And every singleton is open or closed. Hence, G is a topologized graph.

Example: 3.4.1: Let $Bi_{(2,2)}$ be a bipartite graph.



$Bi_{(2,2)}$ topologized graph

$\forall$ V, W $\in$V. V= $\{v_1, v_2\}$; W= $\{w_1, w_2\}$; E= $\{e_1, e_2,\}$

Let v_1, v_2, w_1, w_2 denote the vertices and e_1, e_2 the edges labeled so that $f_G(e_1) = \{v_1, w_2\}$;
$f_G(e_2) = \{v_2, w_1\}$

Let X = $\{v_1, v_2, w_1, w_2, e_1, e_2\}$ be a topological space with the topology defined on

$$\tau = \{X, \phi, \{e_1\}, \{e_2\}, \{v_1, e_1, w_2\}, \{v_2, e_2, w_1\}, \{e_1, e_2\}, \{v_1, e_1, w_2, e_2\} \{v_2, e_2, w_1, e_1\}, \{v_1, e_1\}, \{v_1, e_1, e_2\}, \{v_1, e_1, v_2, e_2, w_1\}\}$$

Here for every $\{x\} \in X$is open or closed.

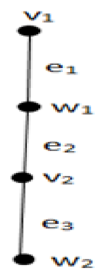
$|\partial(v_1)| = \{w_2\} = 1$; $|\partial(v_2)| = \{w_1\} = 1$;
$|\partial(w_1)| = \{v_2\} = 1$; $|\partial(w_2)| = \{v_1\} = 1$;
$|\partial(e_1)| = \{v_1, w_2\} = 2$; $|\partial(e_2)| = \{v_2, w_1\} = 2$;

The boundary of every vertices and edges are at most two.
Hence $Bi_{(2,2)}$ bipartite graph is topologized graph.

Corollary: 3.4.2: Every tree is a bipartite graph. Then tree is topologized graph.

Proof: Let (X, τ) be a topological space with the topology defined by $V \cup E$. We know every tree is a bipartite graph. Since, a connected acyclic graph is called tree. G is a tree in which each vertex has degree at most two. Therefore, the boundary of every vertices and edges must be at most two. And every singleton is open or closed. Hence, G is a topologized graph**.**

Example: **3.4.3:** Let G be a tree. Then it is a topologized if degree of every vertex is at most two.



Tree topologized graph

$\forall$ V, W $\in$V.V= $\{v_1, v_2\}$ W= $\{w_1, w_2,\}$ E= $\{e_1, e_2, e_3,\}$

Let v_1, v_2, w_1, w_2 denote the vertices and e_1, e_2, e_3 the edges labeled so that

$f_G(e_1) = \{v_1, w_1\}$; $f_G(e_2) = \{v_2, w_1\}$;$f_G(e_3) = \{v_2, w_2\}$ the topology defined on

Let X = $\{v_1, v_2, w_1, w_2, e_1, e_2, e_3,\}$ be a topological space with the topology defined on

$\tau = \{X, \phi, \{v_1\}, \{v_2\}, \{w_1,e_1,e_2,v_2\}, \{v_2,e_3,w_2\}, \{v_2,w_2\}, \{v_1,v_2\}$
$\{v_1,v_2\}, \{v_1,w_1,e_1,e_2,v_2\}, \{v_1,v_2,e_3,w_2\}, \{v_1,v_2,w_2\}, \{v_2,e_3,w_2,w_1,e_1,e_2\}$
$\{w_1,e_1,e_2,v_2,w_2\}, \{v_1,v_2,w_2\}, \{v_1,w_1,e_1,e_2,v_2,w_2\}, \{w_1,w_2,e_1,e_2,v_2,e_3\}$
$\{v_1, w_1, e_1, e_2, v_2, w_2\}, \{w_1, e_1, e_2, v_2\}\}$

Here for every $\{x\} \in X$is open or closed.

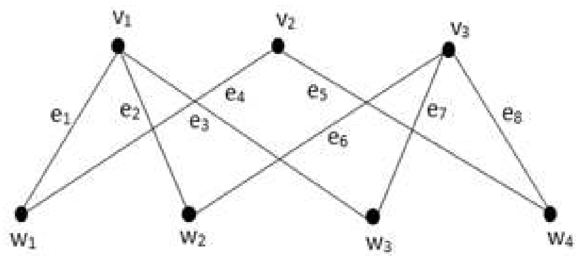
$\forall x \in X,\ |\partial(x)| \leq 2$
$|\partial(v_1)| = \{w_1\} = 1$; $|\partial(v_2)| = \{w_1, w_2\} = 2$;
$|\partial(w_1)| = \{v_1, v_2\} = 2$; $|\partial(w_2)| = \{v_2\} = 1$;
$|\partial(e_1)| = \{v_1, w_1\} = 2$; $|\partial(e_2)| = \{v_2, w_1\} = 2$;
$|\partial(e_3)| = \{v_2, w_2\} = 2$;

Therefore, the boundary of every vertices and edges are at most two. Hence tree is a topologized graph.

Theorem: 3.5: Bipartite graphs contains all the cycles are topologized.

Proof: Let G be a bipartite graph. Let (X, τ) be a topological space with the topology defined by V$\bigcup$ E. $\forall$ V, W $\in$V. Cleary, Every bipartite graph contains only even cycle. It is also known as regular. The cycle of every singleton is open or closed. The boundary of every point is at most two. Therefore bipartite graph contains all the cycles are topologized.

Example: **3.5.1:** Let $Bi_{(3,\,4)}$ be a bipartite graph.

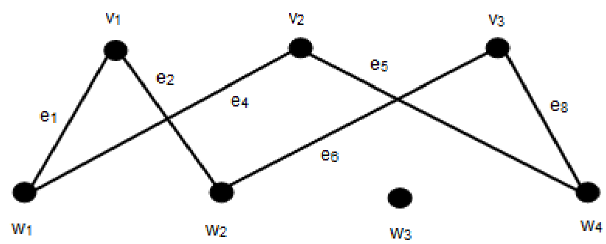


$Bi_{(3,4)}$ bipartite graph

$\forall$ V, W $\in$V. V= {v_1, v_2, v_3} W= {w_1, w_2, w_3, w_4} E= {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8}

Let v_1, v_2,v_3, w_1,w_2 ,w_3, w_4 denote the vertices and e_1, e_2 , $e_3$$e_4$, e_5, e_6, e_7, $e_8$the edges labeled so that $f_G(e_1)$ = {v_1, w_1}; $f_G(e_2)$ = {v_1, w_2}; $f_G(e_3)$ = {v_1, w_3}; $f_G(e_4)$ = {v_2, w_1};$f_G(e_5)$ = {v_2, w_4}; $f_G(e_6)$ = {v_3, w_2}; $f_G(e_7)$ = {v_3, w_3}; $f_G(e_8)$ = {v_3, w_4};

X = {v_1, v_2, v_3, w_1, w_2, w_3, w_4, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8}



C_1 **topologized graph**

Let C_1 = {v_1, e_1, w_1, e_4, v_2, e_5, w_4, e_8, v_3, e_6, w_2, e_2, v_1} be a topological space with the topology defined on τ = {X,ϕ, {e_1},{e_2},{v_1, e_1, w_1, e_4, v_2, e_5, w_4}, {e_8 ,v_3, e_6, w_2, e_2}

{e_8, v_3, w_2, e_2, e_1}, {v_1, e_1, w_1, e_4, v_2, e_5, w_4, e_2}, {e_1, e_2}

Here for every {x} $\in X$is open or closed.

$\forall\ x \in$ X, $|\partial(x)|\leq 2$.

$|\partial(v_1)|$ = {w_1, w_2} = 2; $|\partial(v_2)|$ = {w_1, w_4} = 2; $|\partial(v_3)|$= {w_2, w_4} = 2;

$|\partial(w_1)|$ = {v_1, v_2} =2; $|\partial(w_2)|$ = {v_1, v_3} =2; $|\partial(w_4)|$= {v_2, v_3} = 2;

$|\partial(e_1)|$ = {v_1, w_1} =2; $|\partial(e_2)|$ = {v_1, w_2} =2; $|\partial(e_4)|$= {v_2, w_1} = 2;

$|\partial(e_5)|$ = {v_2, w_4} =2; $|\partial(e_6)|$ = {v_3, w_2} =2; $|\partial(e_8)|$ = {v_3, w_4} =2;

Therefore the boundary of every vertices and edges is at most two.

Hence $Bi_{(3, 4)}$ contains the cycle C_1 is topologized graph.

Similarly, $Bi_{(3, 4)}$ contains the cycles C_2,C_3,C_4,C_5,C_6,C_7,C_8,C_9,C_{10},C_{11} are topologized graph

Theorem: 3.6: $K_{m, n}$ is not a topologized graph (for m, n$\geq$3)

Proof: Let $K_{m, n}$ be a complete bipartite graph. Let (X, τ) be a topological space with the topology defined by $V \bigcup E$. $\forall V, W \in V$.

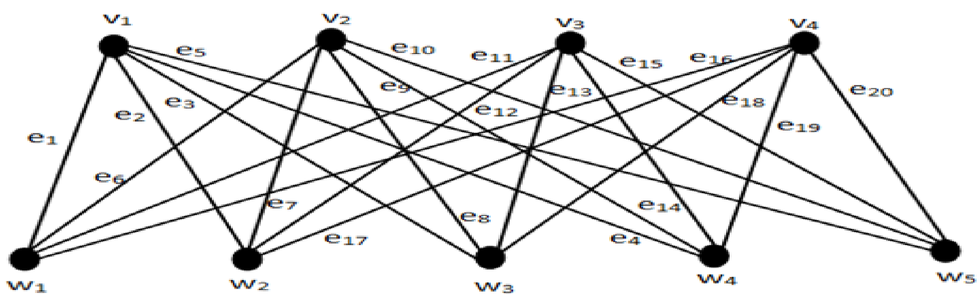
Let m=1 and n=1 for $|X|=3$ since two vertices and one edge. Every singleton is closed (or) open. The boundary of the vertices is two and the boundary of the edge is one. Therefore $K_{1, 1}$ is a topologized graph.

Let m=2 and n=2 for $|X| = 8$, since four vertices and four edges. Every singleton is open (or) closed. The boundary of the vertices and edges is two. Therefore $K_{2, 2}$ is a topologized graph.

Let m=3 and n=3 for $|X|=15$, since six vertices and nine edges. Every singleton is open (or) closed. The boundary of every vertex is three and the edge is two. $K_{3, 3}$ is not satisfied the boundary condition because the boundary of every vertices is less than three. Therefore $K_{3, 3}$ is not topologized. Clearly, if the complete bipartite is not topologized for m, n≥3

Hence $K_{m, n}$ is a not topologized graph. (for m,n≥3)

Example: 3.6.1: Let be $K_{4,5}$a complete bipartite graph.



K 4, 5 non topoogized graph

$\forall$ V, W ∈V.V= $\{v_1, v_2, v_3, v_4\}$; W= $\{w_1, w_2, w_3, w_4, w_5\}$;

E= $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$

Let $v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4, w_5$ denote the vertices and $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}$ the edges labelled so that

$f_G(e_1) = \{v_1, w_1\}$;$f_G(e_2)= \{v_1, w_2\}$; $f_G(e_3) = \{v_1, w_3\}$; $f_G(e_4) = \{v_1, w_4\}$; $f_G(e_5) = \{v_1, w_5\}$;$f_G(e_6) = \{v_2, w_1\}$; $f_G(e_7) = \{v_2, w_2\}$;$f_G(e_8) = \{v_2, w_3\}$; $f_G(e_9) = \{v_2, w_4\}$;$f_G(e_{10}) = \{v_2, w_5\}$; $f_G(e_{11})= \{v_3, w_1\}$; $f_G(e_{12}) = \{v_3, w_2\}$; $f_G(e_{13}) = \{v_3, w_3\}$;$f_G(e_{14}) = \{v_3, w_4\}$; $f_G(e_{15}) = \{v_3, w_5\}$; $f_G(e_{16}) = \{v_4, w_1\}$; $f_G(e_{17}) = \{v_4, w_2\}$;$f_G(e_{18}) = \{v_4, w_3\}$;$f_G(e_{19}) = \{v_4, w_4\}$;$f_G(e_{20}) = \{v_4, w_5\}$;

Let X = $\{v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4, w_5, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}$ be a topological space with the topology defined on

$\tau = \{X, \phi$, $\{e_1\}$,$\{v_1,e_1, w_1,e_2,e_3,e_4,e_5,w_1,w_2,w_3,w_4,w_5\}$,$\{e_{17}\}$,

$\{v_1,e_1, w_1,e_2,e_3,e_4,e_5,w_1,w_2,w_3,w_4,w_5,e_{17}\}$,$\{e_1, e_{17}\}$

$\{v_2,e_6,e_7,e_8,e_9,e_{10},v_3,e_{11},e_{12},e_{13},e_{14},e_{15},v_4,e_{16},e_{17},e_{18},e_{19},e_{20}\}$

$\{v_2,e_6,e_7,e_8,e_9,e_{10},v_3,e_{11},e_{12},e_{13},e_{14},e_{15},v_4,e_{16},e_{17},e_{18},e_{19},e_{20},e_1\}\}$

Here for every $\{x\} \in X$is open or closed.

$\forall x \in X,\ |\partial(x)| \leq 2$

$|\partial(v_1)| = \{w_1, w_2, w_3, w_4, w_5\} = 5$; $|\partial(v_2)| = \{w_1, w_2, w_3, w_4, w_5\} = 5$;

$|\partial(v_3)| = \{w_1, w_2, w_3, w_4, w_5\} = 5$; $|\partial(v_4)| = \{w_1, w_2, w_3, w_4, w_5\} = 5$;

$|\partial(w_1)| = \{v_1, v_2, v_3, v_4\} = 4$; $|\partial(w_2)| = \{v_1, v_2, v_3, v_4\} = 4$;

$|\partial(w_3)| = \{v_1, v_2, v_3, v_4\} = 4$; $|\partial(w_4)| = \{v_1, v_2, v_3, v_4\} = 4$;

$|\partial(w_5)| = \{v_1, v_2, v_3, v_4\} = 4$;

$|\partial(e_1)| = \{v_1, w_1\} = 2$

Similarly the boundary of all edgese_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20} is two. Therefore, the boundary of every edge is two but vertices greater than two. so the boundary condition is not satisfied. Hence $K_{4,5}$ complete bipartite graphs is not a topologized graph**.**

4 Conclusion

The above results we discussed about some new results on bipartite graph. We have applied and verified a new methods topologized graph for star graph for($0<n\leq 2$), bipartite graphs, tree. Further the work is extended to matching and coloring of a bipartite graph.

Competing Interests

Authors have declared that no competing interests exist.

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