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Preliminary Test Stochastic Restricted *r-k* Class Estimator and Preliminary Test Stochastic Restricted *r-d* Class Estimator in Linear Regression Model

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Abstract

In this study two new preliminary test stochastic restricted estimators, a Preliminary Test stochastic restricted r-k class estimator and a Preliminary Test stochastic restricted r-d class estimator, are proposed. The comparison of one estimator over the other is done in the mean square error matrix sense. Further preliminary test stochastic restricted r-k class estimator is compared with r-k class estimator [1] and stochastic restricted r-k class estimator [2]. Similarly preliminary test stochastic restricted r-d class estimator is compared with r-d class estimator [3] and stochastic restricted r-d class estimator [2]. Finally a numerical example and a Monte Carlo simulation study are done to illustrate the theoretical findings of the proposed estimators.

Keywords: Preliminary test estimator, r-d class estimator, r-k class estimator, mean square error matrix

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1 Introduction

Instead of using the Ordinary Least Square Estimator (OLSE), the biased estimators are considered in the regression analysis in the presence of multicollinearity. Some of these are namely the Principal Component Regression Estimator (PCRE) [4], Ridge Estimator (RE) [5], Liu Estimator (LE) [6], r-k class estimator [1], Almost Unbiased Ridge Estimator (AURE) [7], Almost Unbiased Liu Estimator (AULE) [8] and the r-d class estimator [3]. An alternative method to deal with multicollinearity problem is to consider parameter estimation with some restrictions on the unknown parameters, which may be exact or stochastic restrictions. In the presence of stochastic prior information in addition to the sample information, [9] proposed the Mixed Estimator (ME). By replacing OLSE by ME in the RE, LE, AURE and AULE respectively, the Stochastic Mixed Ridge Estimator (SMRE) [10], Stochastic Restricted Liu Estimator (SRLE) [11], Stochastic Restricted Almost Unbiased Ridge Estimator (SRAURE) [12] and Stochastic Restricted Almost Unbiased Liu Estimator (SRAULE) [12] are introduced. Recently, [2] proposed a new stochastic restricted r-k class estimator which is defined by combing the ME and r-k class estimator and a new stochastic restricted r-d class estimator which is defined by combing the ME and r-d class estimator.

When different estimators are available to estimate the unknown parameter, preliminary test estimation procedure is adopted to select a suitable estimator, and it can be also used as another estimator with combining properties of both estimators. The preliminary test approach was first proposed by [13], and then has been studied by many researchers, such as [14,15,16]. By combining OLSE and ME, the Ordinary Stochastic Preliminary Test Estimator (OSPE) was proposed by [15]. Recently, [16] introduced the Preliminary Test Stochastic Restricted Liu Estimator (PTSRLE) by combining the Stochastic Restricted Liu Estimator and Liu Estimator.

In this study, we propose a new Preliminary Test stochastic restricted r-k class estimator which is defined by combing the r-k class estimator and stochastic restricted r-k class estimator and a new Preliminary Test stochastic restricted r-d class estimator which is defined by combing the r-d class estimator and stochastic restricted r-d class estimator. Further the proposed estimators are compared with some biased estimators in the mean square error matrix sense. Also Preliminary Test stochastic restricted r-d class estimator is compared with Preliminary Test stochastic restricted r-k class estimator. Finally a numerical example and a Monte Carlo simulation study are done to illustrate the theoretical findings of the proposed estimators.

2 Model Specification and Estimators

First we consider the multiple linear regression model

$$Y = X\beta + \varepsilon,\ \varepsilon \sim N\left(0, \sigma^2 I\right), \tag{2.1}$$

where Y is an $n \times 1$ observable random vector, X is an $n \times p$ known design matrix of rank p, β is a $p \times 1$ vector of unknown parameters and ε is an $n \times 1$ vector of disturbances.

In addition to sample model (2.1), let us be given some prior information about β in the form of a set of m independent stochastic linear restrictions as follows;

$$r = R\beta + \delta + \upsilon,\ \upsilon \sim N\left(0, \sigma^2\Omega\right) \tag{2.2}$$

where r is an $m\times 1$ stochastic known vector R is a $m\times p$ of full row rank $m \leq p$ with known elements, δ is non zero $m\times 1$ unknown vector and υ is an $m\times 1$ random vector of disturbances and Ω is assumed to be known and positive definite. Further it is assumed that υ is stochastically independent of $\varepsilon.$ i.e. $E\left(\varepsilon\upsilon'\right) = 0$.

The Ordinary Least Squares Estimator (OLSE) for the model (2.1) and the mixed estimator [9] due to a stochastic prior restriction (2.2) are given by

$$\hat{\beta} = S^{-1}X'Y \quad \text{and} \quad \hat{\beta}_m = \hat{\beta} + S^{-1}R'\left(\Omega + RS^{1}R'\right)^{-1}\left(r - R\hat{\beta}\right) \tag{2.3}$$

respectively. where $S = X'X$.

The expectation vector, and the mean square error matrix of $\hat{\beta}$ are given as $E\left(\hat{\beta}\right) = \beta$ and $MSE\left(\hat{\beta}\right) = \sigma^2 S^{-1}$ respectively.

The expectation vector, dispersion matrix, and the mean square error matrix of $\hat{\beta}_m$ are given as $E\left(\hat{\beta}_m\right) = \beta + H\delta$, $D\left(\hat{\beta}_m\right) = \sigma^2 S^{-1} - \sigma^2 G$ and $MSE\left(\hat{\beta}_m\right) = \sigma^2\left(S^{-1} - G\right) + H\delta\delta' H'$ respectively, where,
$G = S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1} RS^{-1}$, $H = S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}$ and $\delta = E\left(r\right) - R\beta$.

When different estimators are available for the same parameter vector β in the linear regression model one must solve the problem of their comparison. Usually as a simultaneous measure of covariance and bias, the mean square error matrix is used, and is defined by

$$MSE(\hat{\beta}, \beta) = E\left[\left(\hat{\beta} - \beta\right)\left(\hat{\beta} - \beta\right)'\right] = D\left(\hat{\beta}\right) + B\left(\hat{\beta}\right)B'\left(\hat{\beta}\right), \tag{2.4}$$

where $D(\hat{\beta})$ is the dispersion matrix, and $B\left(\hat{\beta}\right) = E\left(\hat{\beta}\right) - \beta$ denotes the bias vector. We recall that the Scalar Mean Square Error $SMSE\left(\hat{\beta}, \beta\right) = trace\left[MSE\left(\hat{\beta}, \beta\right)\right]$.

For two given estimators $\hat{\beta}_1$ and $\hat{\beta}_2$, the estimator $\hat{\beta}_2$ is said to be superior to $\hat{\beta}_1$ under the MSEM criterion if and only if

$$M\left(\hat{\beta}_1, \hat{\beta}_2\right) = MSE\left(\hat{\beta}_1, \beta\right) - MSE\left(\hat{\beta}_2, \beta\right) \geq 0 . \tag{2.5}$$

Let us now turn to the question of the statistical evaluation of the compatibility of sample and stochastic information. The classical procedures is to test the hypothesis

$$H_0 : \delta = 0 \text{ against } H_1 : \delta \neq 0 \tag{2.6}$$

under linear model (2.1) and stochastic prior information (2.2).

The Ordinary Stochastic Pre Test Estimator (OSPE) of β [15] is defined as

$$\hat{\beta}_{OSPE} = \begin{cases} \hat{\beta}_m & \text{if } H_0 : \delta = 0 \\ \hat{\beta} & \text{if } H_1 : \delta \neq 0 \end{cases} \tag{2.7}$$

Further, we can write (2.7) as

$$\hat{\beta}_{OSPE} = \hat{\beta}_m I_{\left[0, F_{m,n-p}(\alpha)\right)}\left(F\right) + \hat{\beta} I_{\left[F_{m,n-p}(\alpha), \infty\right)}\left(F\right), \tag{2.8}$$

$$\text{where,} \quad F = \frac{\left(r - R\hat{\beta}\right)' \left(\Omega + RS^{-1}R'\right)^{-1} \left(r - R\hat{\beta}\right)}{m\hat{\sigma}^2} \tag{2.9}$$

which has a non-central $F_{m,n-p,\lambda}$ distribution under $H_1 : \delta \neq 0$, with non-centrality parameter

$$\lambda = \frac{\delta' \left(\Omega + RS^{-1}R'\right)^{-1} \delta}{2\sigma^2} \text{ with } \hat{\sigma}^2 = \frac{\left(Y - X\hat{\beta}\right)' \left(Y - X\hat{\beta}\right)}{n - p}, \tag{2.10}$$

and $I_{\left[0, F_{m,n-p}(\alpha)\right)}\left(F\right)$ and $I_{\left[F_{m,n-p}(\alpha), \infty\right)}\left(F\right)$ are indicator functions which take the value one if F falls in the subscripted interval, and zero otherwise. $F_{m,n-p}\left(\alpha\right)$ is the upper α - level critical value from the central F distribution $F_{m,n-p,0}$.

The expectation vector, dispersion matrix, and the mean square error matrix of $\hat{\beta}_{OSPE}$ are derived by [15], and given by

$$E\left(\hat{\beta}_{OSPE}\right)=\beta+h_{\lambda}\left(2\right)H\delta, \tag{2.11}$$

$$D\left(\hat{\beta}_{OSPE}\right)=\sigma^{2}S^{-1}-\sigma^{2}h_{\lambda}\left(2\right)G+\left[2h_{\lambda}\left(2\right)-h_{\lambda}\left(4\right)-h_{\lambda}^{2}\left(2\right)\right]H\delta\delta'H', \tag{2.12}$$

and

$$MSE\left(\hat{\beta}_{OSPE}\right)=\sigma^{2}S^{-1}-\sigma^{2}h_{\lambda}\left(2\right)G+\left(2h_{\lambda}\left(2\right)-h_{\lambda}\left(4\right)\right)H\delta\delta'H' \tag{2.13}$$

respectively, where, $h_{\lambda}\left(\ell\right)=\Pr\left(\frac{\chi^{2}_{m+\ell,\lambda}}{\chi^{2}_{n-p}}\leq\frac{mF_{m,n-p}(\alpha)}{n-p}\right)$ for $\ell\in\mathrm{N}$.

Now we consider the transformation for model (2.1):

$$y=XTT'\beta+\varepsilon=Z\alpha+\varepsilon \tag{2.14}$$

where $Z=XT$, $\alpha=T'\beta$ and $T=\left(t_{1},t_{2},...,t_{p}\right)=\left(T_{r},T_{p-r}\right)$ is a $p\times p$ orthogonal matrix such that

$$\left(T_{r},T_{p-r}\right)'X'X\left(T_{r},T_{p-r}\right)=\Lambda=\begin{pmatrix}\Lambda_{r} & 0\\ 0 & \Lambda_{p-r}\end{pmatrix}$$

where $0<k\leq p$, $T_{r}=\left(t_{1},t_{2},...,t_{r}\right)$, $T_{p-r}=\left(t_{r+1},t_{r+2},...,t_{p}\right)$, $\Lambda=diag\left(\lambda_{1},\lambda_{2},...,\lambda_{p}\right)$, $\Lambda_{r}=diag\left(\lambda_{1},\lambda_{2},...,\lambda_{r}\right)$, $\Lambda_{p-r}=diag\left(\lambda_{r+1},\lambda_{r+2},...,\lambda_{p}\right)$ and $\lambda_{1}\geq\lambda_{2}\geq...\geq\lambda_{p}>0$ are the eigenvalues of $X'X$. Note that $Z=XT=\left(z_{1},z_{2},...,z_{p}\right)=\left(Z_{r},Z_{r-p}\right)$ is the $n\times p$ matrix of the principal components, where $z_{i}=Xt_{i}$ is the i^{th} principal component. When Z_{p-r} contains principal components corresponding to near zero eigenvalues, Z can be separated as Z_{r} and Z_{p-r}, where Z_{p-r} is to be deleted. Now we can rewrite the model (2.14) as

$$y=XTT'\beta=XT_{r}T_{r}'\beta+XT_{p-r}T_{p-r}'\beta+\varepsilon=Z_{r}\alpha_{r}+Z_{p-r}\alpha_{p-r}+\varepsilon. \tag{2.15}$$

By omitting Z_{p-r}, the OLSE of α_{r} is obtained, and $\hat{\alpha}_{r}=\left(Z_{r}'Z_{r}\right)^{-1}Z_{r}'y$. Then PCRE of β is

$$\hat{\beta}_{PCRE}=T_{r}\left(T_{r}'ST_{r}\right)^{-1}T_{r}'X'y. \tag{2.16}$$

Now the r-k class estimator proposed by [1] and the r-d class estimator proposed by [3] are defined as

$$\hat{\beta}_{rk}\left(r,k\right)=T_r\left(T_r'X'XT_r+kI_r\right)^{-1}T_r'X'y \tag{2.17}$$

and

$$\hat{\beta}_{rd}\left(r,d\right)=T_r\left(T_r'X'XT_r+kI_r\right)^{-1}\left(T_r'X'y+dT_r'\hat{\beta}_r\right) \tag{2.18}$$

respectively.

Followed by [17], the r-k class estimator and the r-d class estimator could be rewritten as follows:

$$\hat{\beta}_{rk}\left(r,k\right)=T_rT_r'\hat{\beta}\left(k\right)=T_rT_r'W_k\hat{\beta}=R_k\hat{\beta} \tag{2.19}$$

and

$$\hat{\beta}_{rd}\left(r,d\right)=T_rT_r'\hat{\beta}\left(d\right)=T_rT_r'F_d\hat{\beta}=H_d\hat{\beta} \tag{2.20}$$

respectively, where $\hat{\beta}\left(k\right)=W_k\hat{\beta}$, $\hat{\beta}\left(d\right)=F_d\hat{\beta}$, $W_k=\left(I+kS^{-1}\right)^{-1}$ for $k\geq 0$, $F_d=\left(S+I\right)^{-1}\left(S+dI\right)$ for $0<d<1$, $R_k=T_rT_r'W_k$ and $H_d=T_rT_r'F_d$.

Note that when $r=p$ we may conclude that $T_rT_r'=I_p$. Hence the estimators $\hat{\beta}_{rk}\left(r,k\right)=W_k\hat{\beta}$ called as RE and $\hat{\beta}_{rd}\left(r,d\right)=F_d\hat{\beta}$ named as LE.

The mean square error matrices of $\hat{\beta}_{rk}\left(r,k\right)$ and $\hat{\beta}_{rd}\left(r,d\right)$ can be obtained as

$$MSE\left[\hat{\beta}_{rk}\left(r,k\right)\right]=\sigma^2R_kS^{-1}R_k'+\left(R_k-I\right)\beta\beta'\left(R_k-I\right)' \tag{2.21}$$

and

$$MSE\left[\hat{\beta}_{rd}\left(r,d\right)\right]=\sigma^2H_dS^{-1}H_d'+\left(H_d-I\right)\beta\beta'\left(H_d-I\right)' \tag{2.22}$$

respectively.

[2] proposed a new stochastic restricted r-k class estimator which is defined by combing the ME and r-k class estimator and a new stochastic restricted r-d class estimator which is defined by combing the ME and r-d class estimator as follows:

$$\hat{\beta}_{SRrk}\left(r,k\right)=T_rT_r'\hat{\beta}\left(k\right)=T_rT_r'W_k\hat{\beta}_m=R_k\hat{\beta}_m \tag{2.23}$$

and

$$\hat{\beta}_{SRrd}\left(r,d\right)=T_rT_r'\hat{\beta}\left(d\right)=T_rT_r'F_d\hat{\beta}_m=H_d\hat{\beta}_m \tag{2.24}$$

respectively.
When $r=p$, the estimator $\hat{\beta}_{SRrk}\left(r,k\right)=W_k\hat{\beta}_m$ called as SMRE and $\hat{\beta}_{SRrd}\left(r,d\right)=F_d\hat{\beta}_m$ named as SRLE.

The mean square error matrices of $\hat{\beta}_{SRrk}\left(r,k\right)$ and $\hat{\beta}_{SRrd}\left(r,d\right)$ can be derived as

$$\begin{aligned} MSE\left[\hat{\beta}_{SRrk}\left(r,k\right)\right]&=\sigma^2R_kS^{-1}R_k'-\sigma^2R_kGR_k' \\ &\quad+\left[\left(R_k-I\right)\beta+R_kH\delta\right]\left[\left(R_k-I\right)\beta+R_kH\delta\right]' \end{aligned} \tag{2.25}$$

and

$$\begin{aligned} MSE\left[\hat{\beta}_{SRrd}\left(r,d\right)\right]&=\sigma^2H_dS^{-1}H_d'-\sigma^2H_dGH_d' \\ &\quad+\left[\left(H_d-I\right)\beta+H_dH\delta\right]\left[\left(H_d-I\right)\beta+H_dH\delta\right]' \end{aligned} \tag{2.26}$$

respectively.

3 Proposed Estimators

[16] proposed the Preliminary Test Stochastic Restricted Liu Estimator (PTSRLE) by combining the LE and SRLE as follows:

$$\tilde{\beta}_{PTSRLE}\left(d\right)=F_d\hat{\beta}_mI_{\left[0,F_{m,n-p}(\alpha)\right)}\left(F\right)+F_d\hat{\beta}\,I_{\left[F_{m,n-p}(\alpha),\infty\right)}\left(F\right)=F_d\hat{\beta}_{OSPE} \tag{3.1}$$

Following [16], we may write the Preliminary Test Stochastic Mixed Ridge Estimator (PTSMRE) by combining RE and SMRE as follows:

$$\tilde{\beta}_{PTSMRE}\left(k\right)=W_k\hat{\beta}_mI_{\left[0,F_{m,n-p}(\alpha)\right)}\left(F\right)+W_k\hat{\beta}\,I_{\left[F_{m,n-p}(\alpha),\infty\right)}\left(F\right)=W_k\hat{\beta}_{OSPE} \tag{3.2}$$

Following [16], we propose a new Preliminary Test stochastic restricted r-k class estimator (PTSRrk) which is defined by combing the r-k class estimator and stochastic restricted r-k class estimator and a new Preliminary Test stochastic restricted r-d class estimator (PTSRrd) which is defined by combing the r-d class estimator and stochastic restricted r-d class estimator as follows:

$$\tilde{\beta}_{PTSRrk}(k) = R_k\left[\hat{\beta}_m I_{\left[0,F_{m,n-p}(\alpha)\right)}(F) + \hat{\beta} I_{\left[F_{m,n-p}(\alpha),\infty\right)}(F)\right] = R_k\hat{\beta}_{OSPE}, \tag{3.3}$$

and

$$\tilde{\beta}_{PTSRrd}(d) = H_d\left[\hat{\beta}_m I_{\left[0,F_{m,n-p}(\alpha)\right)}(F) + \hat{\beta} I_{\left[F_{m,n-p}(\alpha),\infty\right)}(F)\right] = H_d\hat{\beta}_{OSPE} \tag{3.4}$$

respectively.

Now we will see some properties of proposed estimators.

1. When $r = p$, we may conclude that $\tilde{\beta}_{PTSRrk}(k) = \tilde{\beta}_{PTSMRE}(k)$ and $\tilde{\beta}_{PTSRrd}(d) = \tilde{\beta}_{PTSRLE}(d)$.
2. When $r = p$, $k = 0$ and $d = 1$, we may conclude that $\tilde{\beta}_{PTSRrk}(k) = \tilde{\beta}_{PTSRrd}(d) = \hat{\beta}_{OSPE}$.
3. When $\alpha = 0$, we may conclude that $\tilde{\beta}_{PTSRrk}(k) = \hat{\beta}_{SRrk}(r,k)$ and $\tilde{\beta}_{PTSRrd}(d) = \hat{\beta}_{SRrd}(r,d)$.
4. When $\alpha = 1$, we may conclude that $\tilde{\beta}_{PTSRrk}(k) = \hat{\beta}_{rk}(r,k)$ and $\tilde{\beta}_{PTSRrd}(d) = \hat{\beta}_{rd}(r,d)$.

By using (2.11), (2.12) and (2.13), the expectation vector, bias vector, dispersion matrix, and the mean square error matrix of $\tilde{\beta}_{PTSRrk}(k)$ can be shown as follows:

$$E\left[\tilde{\beta}_{PTSRrk}(k)\right] = R_k E\left(\hat{\beta}_{OSPE}\right) = R_k\beta + h_\lambda(2)R_k H\delta, \tag{3.5}$$

$$B\left[\tilde{\beta}_{PTSRrk}(k)\right] = E\left[\tilde{\beta}_{PTSRrk}(k)\right] - \beta = (R_k - I)\beta + h_\lambda(2)R_k H\delta, \tag{3.6}$$

$$\begin{aligned} D\left[\tilde{\beta}_{PTSRrk}(k)\right] &= R_k D\left(\hat{\beta}_{OSPE}\right)R_k' \\ &= \sigma^2 R_k S^{-1}R_k' - \sigma^2 h_\lambda(2)R_k G R_k' + \left[2h_\lambda(2) - h_\lambda(4) - h_\lambda^2(2)\right]R_k H\delta\delta' H' R_k', \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} MSE\left[\tilde{\beta}_{PTSRrk}(k)\right] = {}& \sigma^2 R_k S^{-1}R_k' - \sigma^2 h_\lambda(2)R_k G R_k' \\ &+ \left[2h_\lambda(2) - h_\lambda(4) - h_\lambda^2(2)\right]R_k H\delta\delta' H' R_k' \\ &+ R_k\left[\left(I - R_k^{-1}\right)\beta + h_\lambda(2)H\delta\right]\left[\left(I - R_k^{-1}\right)\beta + h_\lambda(2)H\delta\right]' R_k' \end{aligned} \tag{3.8}$$

respectively.

Similarly the expectation vector, bias vector, dispersion matrix, and the mean square error matrix of $\tilde{\beta}_{PTSRrd}(d)$ can be shown as follows:

$$E\left[\tilde{\beta}_{PTSRrd}(d)\right] = H_d E\left(\hat{\beta}_{OSPE}\right) = H_d\beta + h_\lambda(2) H_d H\delta, \tag{3.9}$$

$$B\left[\tilde{\beta}_{PTSRrd}(d)\right] = (H_d - I)\beta + h_\lambda(2) H_d H\delta, \tag{3.10}$$

$$D\left[\tilde{\beta}_{PTSRrd}(d)\right] = \sigma^2 H'_d S^{-1} H'_d - \sigma^2 h_\lambda(2) H_d G H'_d + \left[2h_\lambda(2) - h_\lambda(4) - h_\lambda^2(2)\right] H_d H\delta\delta' H' H'_d, \tag{3.11}$$

and

$$MSE\left[\tilde{\beta}_{PTSRrd}(d)\right] = \sigma^2 H'_d S^{-1} H'_d - \sigma^2 h_\lambda(2) H_d G H'_d + \left[2h_\lambda(2) - h_\lambda(4) - h_\lambda^2(2)\right] H_d H\delta\delta' H' H'_d + H_d\left[\left(I - H_d^{-1}\right)\beta + h_\lambda(2) H\delta\right]\left[\left(I - H_d^{-1}\right)\beta + h_\lambda(2) H\delta\right]' H'_d \tag{3.12}$$

respectively.

4 Mean Square Error Matrix Comparisons

In this section the Preliminary Test stochastic restricted r-d class estimator will be compared with r-d class estimator and stochastic restricted r-d class estimator and, the Preliminary Test stochastic restricted r-k class estimator will be compared with r-k class estimator and stochastic restricted r-k class estimator in the mean square error matrix sense. Also Preliminary Test stochastic restricted r-d class estimator will be compared with Preliminary Test stochastic restricted r-k class estimator.

4.1 Comparison between $\hat{\beta}_{rk}(r,k)$ and $\tilde{\beta}_{PTSRrk}(k)$

The mean square error matrix difference between $\tilde{\beta}_{PTSRrk}(k)$ and $\hat{\beta}_{rk}(r,k)$ is obtained as

$$MSE\left[\hat{\beta}_{rk}(r,k)\right] - MSE\left[\tilde{\beta}_{PTSRrk}(k)\right] = R_k\left[D + d_1 d'_1 - d_2 d'_2\right] R'_k \tag{4.1}$$

where $D = \sigma^2 h_\lambda(2) G - \xi H\delta\delta' H'$, $\xi = 2h_\lambda(2) - h_\lambda(4) - h_\lambda^2(2)$, $d_1 = \left(I - R_k^{-1}\right)\beta$ and $d_2 = \left(I - R_k^{-1}\right)\beta + h_\lambda(2) H\delta$.

Now the following theorem can be stated.

Theorem 4.1: When $\lambda \le \dfrac{1}{2\left[2-h_\lambda(2)/h_\lambda(4)-h_\lambda(2)\right]}$, the estimator $\tilde{\beta}_{PTSRrk}(k)$ is superior to $\hat{\beta}_{rk}(r,k)$ if and only if $d_2'\left(D+d_1d_1'\right)^{-1}d_2 \le 1$.

Proof: The mean square error matrix difference between $\tilde{\beta}_{PTSRrk}(k)$ and $\hat{\beta}_{rk}(r,k)$ is nonnegative definite matrix if and only if $D+d_1d_1'-d_2d_2' \ge 0$. To find the conditions that $D+d_1d_1'-d_2d_2' \ge 0$ by using lemma 3 (Appendix), we have to show that that D is a nonnegative definite matrix and $d_1, d_2 \in \Re(D)$, where $\Re(.)$ denote the column space of the corresponding matrix.
Note that

$$D=\sigma^2 h_\lambda(2)G-\xi H\delta\delta'H' = \xi\left(\frac{\sigma^2 h_\lambda(2)}{\xi}G-H\delta\delta'H'\right)=\xi D_1$$

where $D_1=\dfrac{\sigma^2 h_\lambda(2)}{\xi}G-H\delta\delta'H'$.

Set $\gamma=\dfrac{\sigma^2 h_\lambda(2)}{\xi}$, $B=G$ and $a=H\delta$.

Note that $G\ge 0$, and the generalized inverse of G is $G^-=SR^+\left(RS^{-1}R'+\Omega\right)\left(R'\right)^+S$. Hence $GG^-H\delta=H\delta$. This implies $H\delta\in\Re(G)$. Then according to lemma 1 (Appendix)

$$\delta'H'G^-H\delta \le \frac{\sigma^2 h_\lambda(2)}{\xi} \tag{4.2}$$

After some straightforward calculations we can easily now show that

$$\delta'H'G^-H\delta=\delta'\left(RS^{-1}R'+\Omega\right)^{-1}\delta \tag{4.3}$$

By substituting this result to (4.2) we can obtain

$$\frac{\delta'\left(RS^{-1}R'+\Omega\right)^{-1}\delta}{2\sigma^2} \le \frac{h_\lambda(2)}{2\xi}. \tag{4.4}$$

Using (2.10), this inequality can be rewritten as

$$\lambda \le \frac{1}{2\left[2-h_\lambda(2)/h_\lambda(4)-h_\lambda(2)\right]} \quad . \tag{4.5}$$

Then according to lemma 1, D_1 is a nonnegative definite matrix, and therefore $D=\xi D_1 \ge 0$ since $\xi \ge 0$.

Now the Moore Penrose inverse of D is obtained by using lemma 2 (Appendix), and is given by

$$D^+ = \frac{1}{\sigma^2 h_\lambda(2)} \times \left[G^+ + \frac{\xi}{\sigma^2 h_\lambda(2) - \xi\delta' H' G^+ H\delta} G^+ H\delta\delta' H' G^+ \right]. \tag{4.6}$$

After some straightforward calculations we can show that

$$\delta' H' G^+ H\delta = 2\sigma^2\lambda . \tag{4.7}$$

Using (4.6) and (4.7) we can easily prove that $DD^+ = I_p$, where I_p is an identity matrix with order $(p \times p)$. This implies that $DD^+ d_1 = d_1$ and $DD^+ d_2 = d_2$. Then we have $d_1 \in \Re(D)$ and $d_2 \in \Re(D)$. To establish condition (a) in the lemma 3, we find $f_{ij} = d_i' D^- d_j$ for $i=1,2,\ j=1,2$ such that

$$f_{11} = \beta'\left(I-R_k^{-1}\right)' D^+ \left(I-R_k^{-1}\right)\beta ,$$

$$f_{22} = \left[\left(I-R_k^{-1}\right)\beta + h_\lambda(2) H\delta\right]' D^+ \left[\left(I-R_k^{-1}\right)\beta + h_\lambda(2) H\delta\right] \text{ and}$$

$$f_{12} = \beta'\left(I-R_k^{-1}\right)' D^+ \left[\left(I-R_k^{-1}\right)\beta + h_\lambda(2) H\delta\right].$$

Note that, instead of D^-, the Moore Penrose inverse D^+ of D is used, since f_{ij} is invariant to the choice of D^-.

Now according to lemma 3 in appendix, $MSE\left[\hat{\beta}_{rk}(r,k)\right] - MSE\left[\tilde{\beta}_{PTSRrk}(k)\right] \ge 0$ if and only if

$$\left(d_1' D^+ d_1 + 1\right)\left(d_2' D^+ d_2 - 1\right) \le \left(d_1' D_1^+ d_2\right)^2 .$$

4.2 Comparison between $\hat{\beta}_{SRrk}(r,k)$ and $\tilde{\beta}_{PTSRrk}(k)$

The mean square error matrix difference between $\tilde{\beta}_{PTSRrk}(k)$ and $\hat{\beta}_{SRrk}(r,k)$ is obtained as

$$MSE\left[\tilde{\beta}_{PTSRrk}(k)\right]-MSE\left[\hat{\beta}_{SRrk}(k)\right]=D_2+d_3d_3'-d_4d_4' \tag{4.8}$$

Where $D_2=\sigma^2\left[1-h_\lambda(2)\right]R_kGR_k'+\xi R_kH\delta\delta'H'R_k'$, $d_3=(R_k-I)\beta+h_\lambda(2)R_kH\delta$ and $d_4=(R_k-I)\beta+R_kH\delta$.

Now we can state the following theorem.

Theorem 4.2: The estimator $\hat{\beta}_{SRrk}(r,k)$ is superior to $\tilde{\beta}_{PTSRrk}(k)$ if and only if $d_4'\left(D_2+d_3d_3'\right)^{-1}d_4\le 1$.

Proof: First we consider the matrix

$$D_2=\sigma^2\left[1-h_\lambda(2)\right]R_kGR_k'+\xi R_kH\delta\delta'H'R_k'.$$

The matrix D_2 can be rewritten as $D_2=R_kM_\ell R_k'$, where $M_\ell=\sigma^2\left[1-h_\lambda(2)\right]G+\xi H\delta\delta'H'$. The matrix $\sigma^2\left[1-h_\lambda(2)\right]G$ is nonnegative definite matrix since $G\ge 0$ and $0\le h_\lambda(2)\le 1$, and the matrix $\xi H\delta\delta'H'$ is positive definite matrix since $H\delta\delta'H'$ is positive definite matrix and $\xi\ge 0$. Therefore the matrix M_ℓ is positive definite matrix, which leads the matrix D_2 is positive definite matrix since $R_k>0$. Now according to lemma 4 (Appendix), the matrix $D_2+d_3d_3'-d_4d_4'\ge 0$ if and only if $d_4'\left(D_2+d_3d_3'\right)^{-1}d_4\le 1$. This completes the proof.

4.3 Comparison between $\hat{\beta}_{rd}(r,d)$ and $\tilde{\beta}_{PTSRrd}(d)$

The mean square error matrix difference between $\tilde{\beta}_{PTSRrd}(d)$ and $\hat{\beta}_{rd}(r,d)$ is obtained as

$$MSE\left[\hat{\beta}_{rd}(r,d)\right]-MSE\left[\tilde{\beta}_{PTSRrd}(d)\right]=H_d\left(D+b_1b_1'-b_2b_2'\right)H_d' \tag{4.9}$$

where, $b_1=\left(I-H_d^{-1}\right)\beta$ and $b_2=\left(I-H_d^{-1}\right)\beta+h_\lambda(2)H\delta$.

Theorem 4.3: When $\lambda \le \dfrac{1}{2\left[2-h_\lambda(2)/h_\lambda(4)-h_\lambda(2)\right]}$, the estimator $\tilde{\beta}_{PTSRrd}(d)$ is superior to $\hat{\beta}_{rd}(r,d)$ if and only if $b_2'\left(D+b_1b_1'\right)^{-1}b_2 \le 1$.

Proof: The mean square error difference between $\tilde{\beta}_{PTSRrd}(d)$ and $\hat{\beta}_{rd}(r,d)$ is nonnegative definite matrix if and only if $D+b_1b_1'-b_2b_2' \ge 0$. We have already proved that $D \ge 0$ and $DD^+ = I_p$. Therefore $DD^+b_1 = b_1$ and $DD^+b_2 = b_2$, which implies that $b_1, b_2 \in \Re(D)$. Now according to lemma 3, $D+b_1b_1'-b_2b_2' \ge 0$ if and only if $\left(b_1'D^+b_1+1\right)\left(b_2'D^+b_2-1\right) \le \left(b_1'D_1^+b_2\right)^2$. This completes the proof.

4.4 Comparison between $\hat{\beta}_{SRrd}(r,d)$ and $\tilde{\beta}_{PTSRrd}(d)$

The mean square error matrix difference between $\tilde{\beta}_{PTSRrd}(d)$ and $\hat{\beta}_{SRrd}(r,d)$ is obtained as

$$MSE\left[\tilde{\beta}_{PTSRrd}(d)\right]-MSE\left[\hat{\beta}_{SRrd}(d)\right]=B_1+b_3b_3'-b_4b_4' \tag{4.10}$$

where $B_1=\sigma^2\left[1-h_\lambda(2)\right]H_dGH_d'+\xi H_dH\delta\delta'H'H_d'$, $b_3=\left(H_d-I\right)\beta+h_\lambda(2)H_dH\delta$ and $b_4=\left(H_d-I\right)\beta+H_dH\delta$.

Now the following theorem can be stated.

Theorem 4.4: The estimator $\hat{\beta}_{SRrd}(r,d)$ is superior to $\tilde{\beta}_{PTSRrd}(d)$ if and only if $b_4'\left(B_1+b_3b_3'\right)^{-1}b_4 \le 1$.

Proof: The matrix B_1 can be rewritten as $B_1 = H_dM_\ell H_d'$. We have already proved that M_ℓ is positive definite matrix. Therefore the matrix B_1 is positive definite matrix since $H_d > 0$. Now according to lemma 1, the matrix $B_1+b_3b_3'-b_4b_4' \ge 0$ if and only if $b_4'\left(B+b_3b_3'\right)^{-1}b_4 \le 1$. This completes the proof.

4.5 Comparison between $\tilde{\beta}_{PTSRrd}(d)$ and $\tilde{\beta}_{PTSRrk}(k)$

The mean square error matrix between $\tilde{\beta}_{PTSRrd}(d)$ and $\tilde{\beta}_{PTSRrk}(k)$ is given as

$$MSE\left[\tilde{\beta}_{PTSRrk}(k)\right]-MSE\left[\tilde{\beta}_{PTSRrd}(d)\right]=\vartheta+d_3d_3'-b_3b_3' \tag{4.11}$$

where $\vartheta=R_kM_xR_k'-H_dM_xH_d'$ and $M_x=\sigma^2S^{-1}-\sigma^2h_\lambda(2)G+\xi H\delta\delta'H'$.

Theorem 4.5: When maximum eigenvalues of $\left(H_dM_xH_d'\right)\left(R_kM_xR_k'\right)^{-1}$ is less than one, the estimator $\tilde{\beta}_{PTSRrd}(d)$ is superior to $\tilde{\beta}_{PTSRrk}(k)$ if and only if $b_3'\left(\vartheta+d_3d_3'\right)^{-1}b_3\le 1$.

Proof: First we consider $M_x=\sigma^2S^{-1}-\sigma^2h_\lambda(2)G+\xi H\delta\delta'H'$.Note that $\sigma^2S^{-1}-\sigma^2h_\lambda(2)G\ge\sigma^2\left(S^{-1}-G\right)$ since $0\le h_\lambda(2)\le 1$. But $S^{-1}-G=\left(S+R\Omega^{-1}R'\right)^{-1}\ge 0$.

Therefore $\sigma^2S^{-1}-\sigma^2h_\lambda(2)G\ge 0$. Since $\sigma^2S^{-1}-\sigma^2h_\lambda(2)G\ge 0$ and $\xi H\delta\delta'H'>0$, the matrix $M_x>0$. Therefore $H_dM_xH_d'$ and $R_kM_xR_k'$ are positive definite matrices since $H_d>0$ and $R_k>0$ respectively. So that, according to lemma 5, $\vartheta=R_kM_xR_k'-H_dM_xH_d'>0$ if and only if maximum eigenvalues of $\left(H_dM_xH_d'\right)\left(R_kM_xR_k'\right)^{-1}$ is less than one. Applying lemma 4, we can easily prove that the estimator $\tilde{\beta}_{PTSRrd}(d)$ is superior to $\tilde{\beta}_{PTSRrk}(k)$ if and only if $b_3'\left(\vartheta+d_3d_3'\right)^{-1}b_3\le 1$. This completes the proof.

Note: In the above theorem, when $\alpha=0$ and $\alpha=1$, we can obtain the superiority conditions of the Stochastic Restricted r-k class estimator over the Stochastic Restricted r-d class estimator, and the superiority conditions of the r-k class estimator over the r-d class estimator respectively.

5 Illustration of Theoretical Results

5.1 Numerical Example

To illustrate our theoretical results in this section we consider the data set which was discussed in [18] and later considered by [10,19,20,21,22,23]. Table 5.1 gives Total National Research and Development Expenditures-as a Percent of Gross National Product by Country: 1972-1986. It represents the relationship between the dependent variable y the percentage spent by the United States and the dependent variables x_1, x_2, x_3 and x_4. The variable x_1 represents the percent spent by France, x_2 that spent by West Germany, x_3 that spent by Japan, and x_4 that spent by the former Soviet Union.

Table 5.1 Total national research and development expenditures-as a percent of gross national product by country: 1972-1986

Year	y	x_1	x_2	x_3	x_4
1972	2.3	1.9	2.2	1.9	3.7
1975	2.2	1.8	2.2	2.0	3.8
1979	2.2	1.8	2.4	2.1	3.6
1980	2.3	1.8	2.4	2.2	3.8
1981	2.4	2.0	2.5	2.3	3.8
1982	2.5	2.1	2.6	2.4	3.7
1983	2.6	2.1	2.6	2.6	3.8
1984	2.6	2.2	2.6	2.6	4.0
1985	2.7	2.3	2.8	2.8	3.7
1986	2.7	2.3	2.7	2.8	3.8

We assemble the data in the matrix form as follows:

$$X = \begin{pmatrix} 1.9 & 2.2 & 1.9 & 3.7 \\ 1.8 & 2.2 & 2.0 & 3.8 \\ 1.8 & 2.4 & 2.1 & 3.6 \\ 1.8 & 2.4 & 2.2 & 3.8 \\ 2.0 & 2.5 & 2.3 & 3.8 \\ 2.1 & 2.6 & 2.4 & 3.7 \\ 2.1 & 2.6 & 2.6 & 3.8 \\ 2.2 & 2.6 & 2.6 & 4.0 \\ 2.3 & 2.8 & 2.8 & 3.7 \\ 2.3 & 2.7 & 2.8 & 3.8 \end{pmatrix}, \quad y = \begin{pmatrix} 2.3 \\ 2.2 \\ 2.2 \\ 2.3 \\ 2.4 \\ 2.5 \\ 2.6 \\ 2.6 \\ 2.7 \\ 2.7 \end{pmatrix}$$

The four column of the 10×4 matrix X comprise the data on x_1, x_2, x_3 and x_4 respectively, and y is the response variable. Note that the eigen values of S are $\lambda_1 = 302.9626$, $\lambda_2 = 0.7283$, $\lambda_3 = 0.0447$ and $\lambda_4 = 0.0345$ and the condition number of X is approximately 8781.53. This implies the existence of multicollinearity in the data set. The OLSE is given by

$$\hat{\beta}_{OLSE} = S^{-1}X'y = (0.6455, 0.0896, 0.1436, 0.1526)'$$

with $MSE\left(\hat{\beta}_{OLSE}, \beta\right) = 0.0808$ and $\hat{\sigma}^2 = 0.0015$.

Consider the following stochastic restrictions

$r = R\beta + \delta + \nu$ where $R = (1, -2, -2, -2)'$, $r = 1$ and $\nu \sim N\left(0, \hat{\sigma}^2 = 0.0015\right)$.

We select the significance level $\alpha = 0.05$. Following [24] we choose the number of the principal components $r = 3$. Fig. 1, Fig. 2 and Fig. 3 are obtained by using the equations (2.21), (2.22), (2.25), (2.26), (3.8) and (3.12) for different shrinkage parameters *d* and *k* values selected from the interval (0, 1).

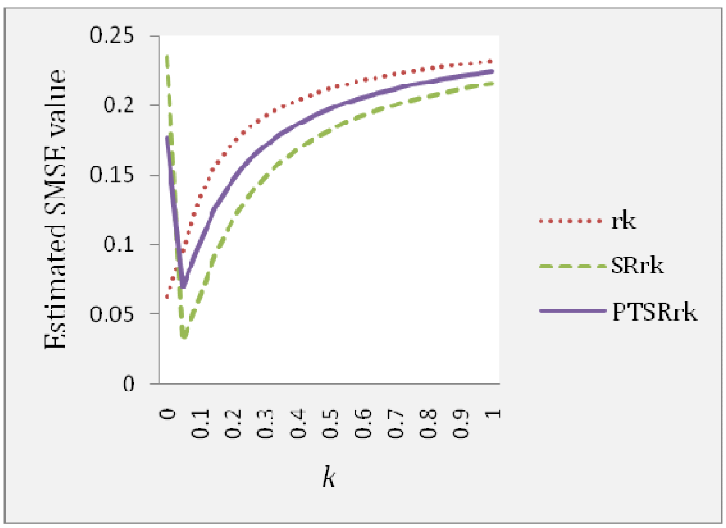

Fig. 1. Estimated SMSE value of rk, SRrk and PTSRrk

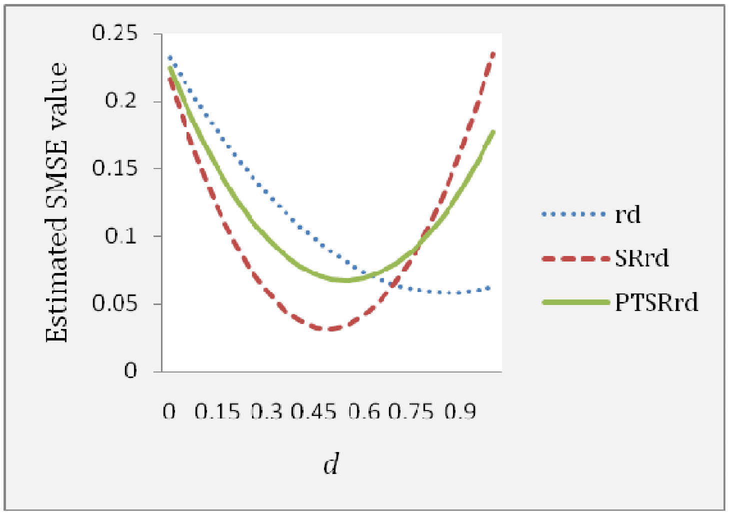

Fig. 2. Estimated SMSE value of rd, SRrd and PTSRrd

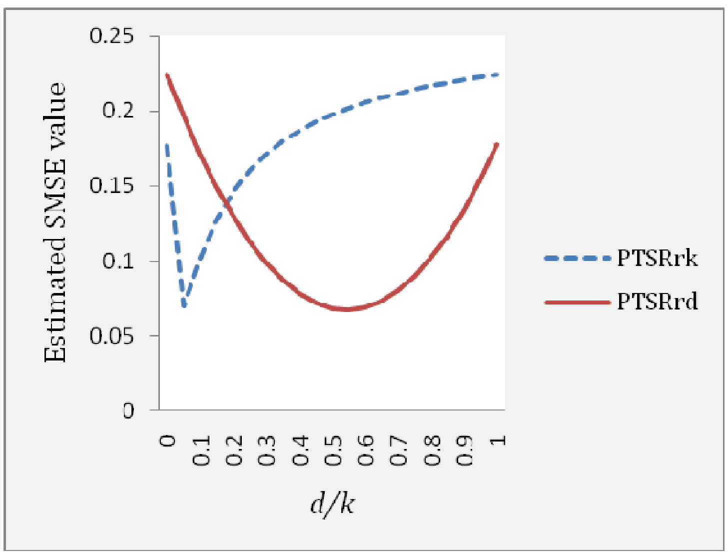

Fig. 3. Estimated SMSE value of PTSRrk and PTSRrd

From Fig. 1, when k is small the PTSRrk has the smallest SMSE than stochastic restricted r-k class estimator. When k is large, the PTSRrk has the smallest SMSE than r-k class estimator. From Fig. 2, when d is small the PTSRrd has the smallest SMSE than r-d class estimator. When d is large, the PTSRrd has the smallest SMSE than stochastic restricted r-d class estimator. From Fig. 3, when d/k is small, the PTSRrk has the smallest SMSE than PTSRrd, the situation is reversed when *d/k* is large. From Fig. 1, Fig. 2 and Fig. 3, we can say that no estimator is always superior to other estimators.

5.2 Simulation Study

To illustrate the statistical behavior of our proposed estimators, we perform a Monte Carlo Simulation study by considering different levels of multicollinearity. Following [25] we generate explanatory variables as follows:

$$x_{ij} = \left(1-\rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1},\ i = 1,2,...,n,\ \ j = 1,2,..., p,$$

where z_{ij} is an independent standard normal pseudo random number, and ρ is specified so that the theoretical correlation between any two explanatory variables is given by ρ^2. A dependent variable is generated by using the equation.

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \varepsilon_i,\ i = 1,2,...,n,$$

where ε_i is a normal pseudo random number with mean zero and variance σ_i^2. [26] have noted that if the MSE is a function of σ^2 and β, and if the explanatory variables are fixed, then subject to the constraint $\beta'\beta = 1$, the MSE is minimized when β is the normalized eigenvector corresponding to the largest eigenvalue of the $X'X$ matrix. In this study we choose the normalized eigenvector corresponding to the largest eigenvalue of $X'X$ as the coefficient vector β, $n = 50$, $p = 4$ and $\sigma^2 = 1$. Two different sets of correlations are considered by selecting the values as $\rho = 0.8$ 0.8 and 0.9, and the significance level is taken as $\alpha = 0.05$. Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8 and Fig. 9 are obtained by using the equations (2.21), (2.22), (2.25), (2.26), (3.8) and (3.12) for different shrinkage parameters *d* and *k* values selected from the interval (0, 1).

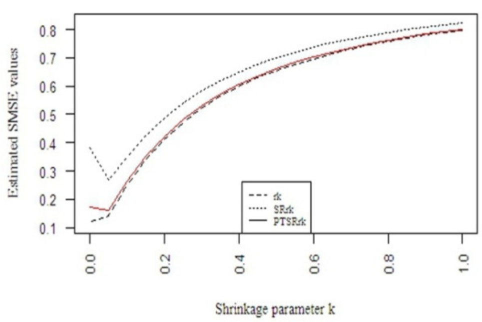

Fig. 4. Estimated SMSE value of rk, SRrk and PTSRrk for $\rho = 0.8$

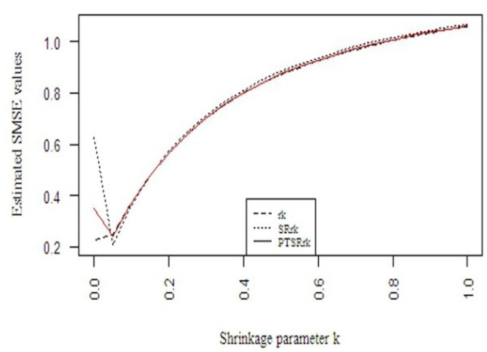

Fig. 5. Estimated SMSE value of rk, SRrk and PTSRrk for $\rho = 0.9$

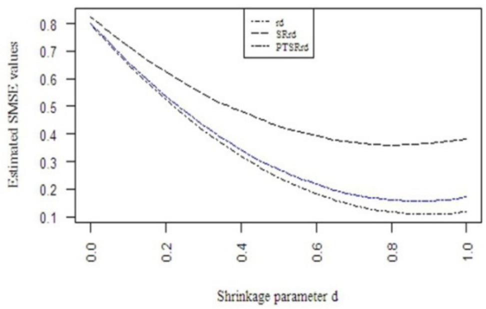

Fig. 6. Estimated SMSE value of rd, SRrd and PTSRrd for $\rho = 0.8$

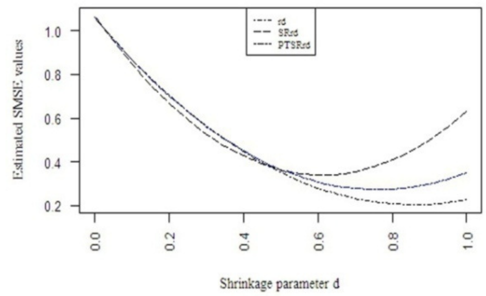

Fig. 7. Estimated SMSE value of rd, SRrd and PTSRrd for $\rho = 0.9$

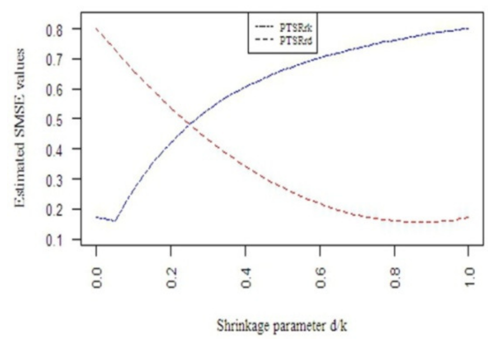

Fig. 8. Estimated SMSE value of PTSRrk and PTSRrd for $\rho = 0.8$

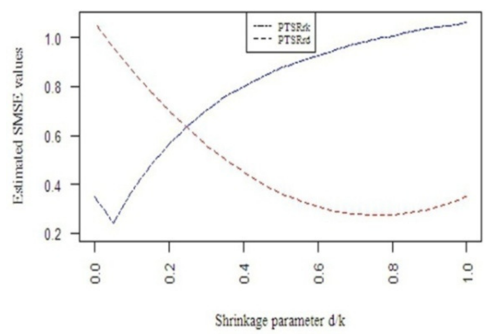

Fig. 9. Estimated SMSE value of PTSRrk and PTSRrd for $\rho = 0.9$

Based on Fig. 4, r-k class estimator and PTSRrk have smallest SMSE than stochastic restricted r-k class estimator. According to Fig.5, there is no big difference in the SMSE except k=0. From Fig.6 and Fig.7, we can say that, when d is large the estimator r-d class estimator has the smallest SMSE than PTSRrd. From Fig.8 and Fig.9 we can notice that when d or k is small, the PTSRrk has the smallest SMSE than PTSRrd, the situation is reversed when d or k is large.

Remark: [27] proposed the Generalized Preliminary Test Stochastic Restricted Estimator (GPTSRE) as follows:

$$\tilde{\beta}_{GPTSRE} = A_{(j)}\hat{\beta}_m I_{\left[0,F_{m,n-p}(\alpha)\right)}(F) + A_{(j)}\hat{\beta}\, I_{\left[F_{m,n-p}(\alpha),\infty\right)}(F) = A_{(j)}\hat{\beta}_{OSPE}$$

where $A_{(j)}$ is a positive definite matrix.

Note that when $A_{(j)} = R_k$, $\tilde{\beta}_{GPTSRE}$ gives the Preliminary Test stochastic restricted r-k class estimator (PTSRrk), and when $A_{(j)} = H_d$, it gives the Preliminary Test stochastic restricted r-d class estimator (PTSRrd).

6 Conclusion

In this study, we propose a new Preliminary Test stochastic restricted r-k class estimator which is defined by combing the r-k class estimator and stochastic restricted r-k class estimator and a new Preliminary Test stochastic restricted r-d class estimator which is defined by combing the r-d class estimator and stochastic restricted r-d class estimator. Further the proposed estimators are compared with some biased estimators in the mean square error matrix sense. Also Preliminary Test stochastic restricted r-d class estimator is compared with Preliminary Test stochastic restricted r-k class estimator. Finally a numerical example and a Monte Carlo simulation study are done to illustrate the theoretical findings of the proposed estimators. Based on the theoretical findings and numerical illustration, we can conclude that no estimator is always superior to other estimators.

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Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

Lemma 1: [28]

Suppose B is a symmetric real $(n \times n)$ matrix, a is an $(n \times 1)$ real vector and γ is a positive real number. Then the following two properties are equivalent

a) $\gamma B - aa'$ is nonnegative definite (n.n.d)

b) B is n.n.d, $a \in \Re(B)$ and $a'B^{-}a \leq \gamma$.

Lemma 2: [29]

Let A be a symmetric $(n \times n)$ matrix, and let a, a_1, and a_2 be $(n \times 1)$ vectors. Suppose that

a) $a \in \Re(A)$,and the real numbers ϕ and ψ satisfy $\phi \neq 0$ and $\phi + \psi a'A^{+}a \neq 0$. Then we have the identity

$$\left[\phi A + \psi aa'\right]^{+} = \frac{1}{\phi}\left[A^{+} - \frac{\psi}{\phi + \psi a'A^{+}a} A^{+}aa'A^{+}\right]$$

b) $a_j \in \Re(A)$, $j = 1,2$, and the real number ρ satisfies $1 + \rho a_1'A^{+}a_1 \neq 0.$ Then we have $a_2 \in \Re(A + \rho a_1 a_1')$.

Lemma 3: [30]

Let C be a nonnegative definite matrix and c_1, c_2 be linearly independent vectors. Furthermore for some generalized inverse C^{-} of C, let $f_{ij} = c_i'C^{-}c_j$; $i = 1,2,\ j = 1,2$ and let

$$s = \frac{c_2'(I - CC^{-})'(I - CC^{-})c_2}{c_1'(I - CC^{-})(I - CC^{-})c_1}$$

where $c_1 \in \Re(C)$ and $\Re(.)$ denote the column space of the corresponding matrix. Then we have $C + c_1c_1' - c_2c_2' \geq 0$ if and only if

a) $c_1 \in \Re(C), c_2 \in \Re(C)$ and $(f_{11} + 1)(f_{22} - 1) \leq f_{12}^2$ or

b) $c_1 \notin \Re(C), c_2 \in \Re(C, c_1)$ and $(c_2 - sc_1)'C^{-}(c_2 - sc_1) \leq 1 - s^2$

and all expressions in (a) and (b) are independent of the choice of C^{-}.

Lemma 4: [31]

Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be two linear estimator of β. Suppose that $D = D(\hat{\beta}_1) - D(\hat{\beta}_2)$ is positive definite then $\Delta = MSE(\hat{\beta}_1) - MSE(\hat{\beta}_2)$ is nonnegative definite if and only if $b_2'(D + b_1 b_1')^{-1} b_2 \leq 1$, where b_j denotes the bias vector of $\hat{\beta}_j$, $j = 1, 2$.

Lemma 5: [32]

Let $n \times n$ matrices $M > 0$, $N > 0$ (or $N \geq 0$), then $M > N$ if and only if $\lambda_1\left(NM^{-1}\right) < 1$. where $\lambda_1\left(NM^{-1}\right)$ is the largest eigenvalue of the matrix NM^{-1}.

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