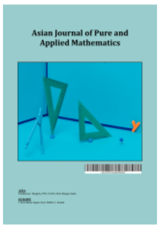

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Finite Anti-Plane Shear Deformation of a Neo-Hookean Material Using Monge Method of Solution

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Authors' contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Finite deformation of an incompressible hollow Neo-Hookean material under anti-plane shear is investigated. The problem is converted from Cartesian co-ordinate to cylindrical polar co-ordinate since the problem is better handled in cylindrical polar co-ordinate. The analysis produces an elliptic second order partial differential equation which sought for Monge Method of solution for the determination of displacement and stresses. Boundary value conditions are set up in determining the contacts of integration involved in the solution. Finally a closed solution for the displacement and stresses at any cross section of the cylinder is achieved.

Keywords: Anti-plane shear; displacement; deformed radius; shear stresses; monge mothed.

1 Introduction

The major problem in the analysis of solids deforming under anti-plane shear is difficult to achieve closed form solutions to the boundary value problems resulting from the analysis. Anti-plane shear deformations are one of the simplest classes of deformations that solids can undergo. Anti-plane shear involve

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deformations of a cylindrical region in such a way that the displacement of each particle is parallel to the axial direction and independent of its axial coordinate. For the case of linear isotropic material like neo Hookean material, the strain energy function W =W(I_1) where I_1 is the first principal invariant of the left Cauchy Green Strain tensor $B = FF^T$, F being the gradient of deformation. The absent of I_2 in the strain energy function enables us to achieve analytic solution with the boundary value problem. When we have linear isotropic elasticity, the three dimensional equations can be reduced to a single two dimensional second order equation which is the axial component of the equation for a single unknown usually denoted as the out of plane displacement. Erumaka [1] in one of his works investigated on "finite anti plane shear deformation of a class of Ogden containing a line crack". Erumaka and Onugha [2] analyzed "the stresses and deformation in a Neo-Hookean Half space deforming under Anti plane shear loading". Polygnone and Horgan [3] worked on "Axisymmetric finite Anti plane shear of compressible Non linearly elastic circular Tubes". Ejike and Erumaka [4] investigated "finite deformation of a rotating circular cylinder of Blatz-ko material". Simmand and Warne [5] studied "Azimuthal shear of compressible non-linaerly Elastic orthotropic Tubes of finite Extent". Horgan and Giuseppe [6] analyzed "Anti-plane shear deformations for non-Gaussian isotropic, incompressible hyperelastic materials". Also Horgan and Giuseppe [7] extended "the work to Superposition of Generalized plane strain on anti-plane shear deformations in isotropic incompressible Hyperelastic". Lejla [8] did work on "Two parametric analysis of Anti-plane shear deformation of a coated Elastic Half-Space" and Horgan [9] investigated on "Anti-plane shear deformations in linear and non linear solid mechanics". Moreover Yang [10] investigated "Asymptotic solution to Axisymmetric indention of a compressible elastic thin film". Darijani and Bahremen [11] used "polynomial hyperelastic models to obtain a closed form solution for analyses of rubbery solid circular cylinder". Robert et al [12] with "the use of neo-Hookean and the Mooney–Rivlin models found the strain energy function for isotropic incompressible solids demonstrating a linear relationship between shear stress and amount of shear, and between torque and amount of twist, when subject to large simple shear or torsion deformations". Merodio and Ogden [13] proposed "a new example of the solution to the finite deformation boundary value problem for a residually stressed elastic body and combined extension, inflation and torsion of a circular cylindrical tube subject to radial and circumferential residual stress. The isochoric deformation consisted of axial extension, radial inflation and superimposed torsion which is formulated for a general elastic strain energy function. Two simple strain energy functions incorporating radial stress were used and the integral were evaluated to give close form expressions for pressure, axial load and torsional movement. In addition to works done in elasticity on cylindrical materials". Anani and Gholamhosein [14] worked on "spherical material, Stress analysis of thick pressure vessel composed of incompressible hyperelastic materials where Neo Hookean strain energy function was used to determine the stress and displacement of spherical shell that is axisymmetric radially deformed under internal and external pressure. Exact solutions were derived for stress and stretch in a thick hyperelastic spherical shell and the effect of the structure parameter for different examples was discussed". Shearer et al. [15] analyzed "Torsional wave propagation in a pre-stressed hyperelastic annular circular cylinder".

Grine et al. [16] worked on Elastic machines, a non standard use of the axial shear of linear transversely isotropic elastic cylinders. Marco and Giuseppe [17] investigated on superposing plane strain on anti-plane shear deformations in a special class of fibre-reforced incompressible hyperelastic materials. In this present paper we sought for analytic solution using Monge method of solution for the determination of stresses and displacement across a hollow cylinder made of Neo-hookean material.

2 Governing Equations

Let the deformation of an isotropic, homogenous, incompressible, elastic material that takes the_point (X_1, X_2, X_3) of the initial configuration to the point (x_1, x_2, x_3) of the deformed configuration in anti-plane shear be

$$x_1 = X_1 \, , x_2 = X_2 \, , x_3 = X_3 + w(X_1, X_2) \tag{1}$$

$w(X_1, X_2)$ is called out of plane displacement or anti-plane displacement and w depends continuously on X_1 and X_2 coordinates and twice differentiable function of X_1 and X_2 at every point of D and its deformed image D^*.

The deformation gradient tensor F for equation (1) is given as

$$\overline{\mathrm{F}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ w_1 & w_2 & 1 \end{bmatrix} \tag{2}$$

where $\frac{\partial w}{\partial X_1} = w_1$, $\frac{\partial w}{\partial X_2} = w_2$

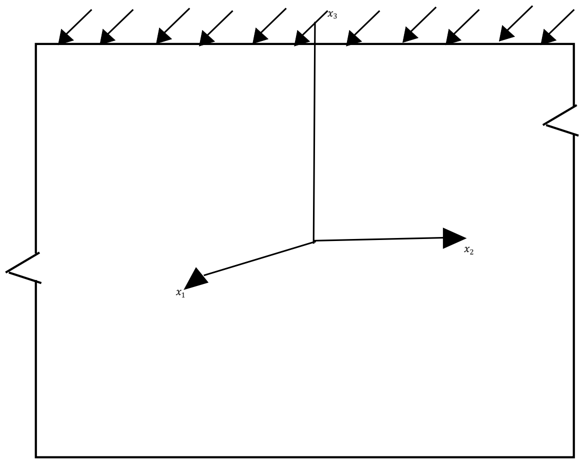

Fig. 1. Anti-plane shear deformation

The Left Cauchy-Green deformation gradient tensor B associated with (1) is given as

$$\overline{\mathrm{B}} = \overline{\mathrm{F}}\overline{\mathrm{F}}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ w_1 & w_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & w_1 \\ 0 & 1 & w_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\mathrm{B}} = \begin{bmatrix} 1 & 0 & w_1 \\ 0 & 1 & w_2 \\ w_1 & w_2 & 1 + {w_1}^2 + {w_2}^2 \end{bmatrix} \tag{3}$$

$$\overline{\mathrm{B}}^{-I} = \begin{bmatrix} 1 + {w_1}^2 & w_1 w_2 & -w_1 \\ w_1 w_2 & 1 + {w_1}^2 & -w_2 \\ -w_1 & -w_2 & 1 \end{bmatrix}$$

$$\overline{\mathrm{B}}^2 = \overline{\mathrm{B}}.\overline{\mathrm{B}} = \begin{bmatrix} 1 + {w_1}^2 & w_1 w_2 & w_1(2 + {w_1}^2 + {w_2}^2) \\ w_1 w_2 & 1 + w_2 & w_2(2 + {w_1}^2 + {w_2}^2) \\ w_1(2 + {w_1}^2 + {w_2}^2) & w_2(2 + {w_1}^2 + {w_2}^2) & 1 + 3{w_1}^2 + 3{w_2}^2 + {w_1}^4 + {w_2}^4 + 2{w_1}^2{w_2}^2 \end{bmatrix}$$

The Left Cauchy-Green tensor is a second order tensor and symmetric, then it has three principal strain invariants I_1 , I_2 , and I_3 given as

$$\mathrm{I}_1 = \text{trace } \overline{\mathrm{B}},\ \mathrm{I}_2 = \frac{1}{2}[(tr\overline{\mathrm{B}})^2 - tr\overline{\mathrm{B}}^2] = \text{trace } \overline{\mathrm{B}}^{-I},\ \mathrm{I}_3 = \det \overline{\mathrm{B}}$$

$$I_1 = 3 + {w_1}^2 + {w_2}^2, I_2 = 3 + {w_1}^2 + {w_2}^2 = I_1 , I_3 = 1 \tag{4}$$

The third strain invariant shows that the material is incompressible, since $I_3 = 1$

Using the Neo-Hookean strain energy function material which is given by

$$W = \frac{\mu}{2}(I_1 - 3) \tag{5}$$

where μ is the shear modulus

$$W_1 = \frac{\partial W}{\partial I_1} = \frac{\mu}{2}\ W_2 = \frac{\partial W}{\partial I_2} = 0 \tag{6}$$

3 Stress Tensor $\boldsymbol{\tau}$

The stress tensor for incompressible material is given by

$$\tau = -\rho I + 2W_1 B - 2W_2 B^{-1} \tag{7}$$

where I is the unit tensor and ρ is the hydrostatic pressure

$$\tau = \begin{bmatrix} -\rho & 0 & 0 \\ 0 & -\rho & 0 \\ 0 & 0 & -\rho \end{bmatrix} + \begin{bmatrix} \mu & 0 & \mu w_1 \\ 0 & \mu & \mu w_2 \\ \mu w_1 & \mu w_2 & \mu + \mu {w_1}^2 + \mu {w_2}^2 \end{bmatrix}$$

Simplifying further, we have

$$\tau = \begin{bmatrix} -\rho + \mu & 0 & \mu w_1 \\ 0 & -\rho + \mu & \mu w_2 \\ \mu w_1 & \mu w_2 & -\rho + \mu + \mu {w_1}^2 + \mu {w_2}^2 \end{bmatrix} \tag{8}$$

Representing stress tensor in Cartesian co-ordinate, form we have

$$\tau = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \tag{9}$$

Comparing (8) and (9) we obtain

$$\tau_{xx} = -\rho + \mu \tag{10a}$$

$$\tau_{xy} = \tau_{yx} = 0 \tag{10b}$$

$$\tau_{xz} = \tau_{zx} = \mu w_1 \tag{10c}$$

$$\tau_{yy} = -\rho + \mu \tag{10d}$$

$$\tau_{yz} = \tau_{zy} = \mu w_2 \tag{10e}$$

$$\tau_{zz} = -\rho + \mu + \mu {w_1}^2 + \mu {w_2}^2 \tag{10f}$$

Here we need to transform the field equations to cylindrical polar coordinates (r, θ, z) since the problem is better handled in cylindrical polar co-ordinates.

The Cartesian co-ordinate is related to cylindrical polar co-ordinate by the relation given below

$$X_1 = rcos\theta,\ X_2 = rsin\theta\ , X_3 = Z$$

$$r = ({X_1}^2 + {X_2}^2)^{\frac{1}{2}}\,\theta = tan^{-1}\left(\frac{X_2}{X_1}\right)$$

That is from w$(X_1,\, X_2)$ $to\ w = w(r,\theta)$

From total differentiation

$$\partial w(r,\theta) = \frac{\partial w(r,\theta)}{\partial r}\partial r + \frac{\partial w(r,\theta)}{\partial \theta}\partial\theta$$

since r and θ are functions of X_1 and X_2 then we have

$$\frac{\partial w(r,\theta)}{\partial X_1} = \frac{\partial w(r,\theta)}{\partial r}\frac{\partial r}{\partial X_1} + \frac{\partial w(r,\theta)}{\partial \theta}\frac{\partial \theta}{\partial X_1} \tag{11}$$

$\frac{\partial r}{\partial X_1} = cos\theta$ and $\frac{\partial \theta}{\partial X_1} = \frac{-sin\,\theta}{r}$

Then (11) becomes

$$w_1 = \ w_r cos\theta - w_\theta \frac{sin\,\theta}{r} \tag{12}$$

Squaring both sides of (12)

$$(w_1)^2 = \ \cos^2\theta(w_r)^2 - \frac{2}{r}cos\theta sin\theta w_r w_\theta + \frac{1}{r^2}\ \sin^2\theta(w_\theta)^2$$

$$\frac{\partial w(r,\theta)}{\partial X_2} = \frac{\partial w(r,\theta)}{\partial r}\frac{\partial r}{\partial X_2} + \frac{\partial w(r,\theta)}{\partial \theta}\frac{\partial \theta}{\partial X_2} \tag{13}$$

$$\frac{\partial r}{\partial X_2} = sin\theta\ and\ \frac{\partial \theta}{\partial X_2} = \frac{cos\theta}{r}$$

$$w_2 = w_r sin\theta + w_\theta \frac{cos\theta}{r} \tag{14}$$

Squaring both sides of (14)

$$(w_2)^2 = \ \sin^2\theta(w_r)^2 + \frac{2}{r}cos\theta sin\theta w_r w_\theta + \frac{1}{r^2}\ \cos^2\theta(w_\theta)^2$$

From (10f) that is $\tau_{zz} = -\rho + \mu + \mu {w_1}^2 + \mu {w_2}^2$

$$\begin{aligned} -\rho + \mu(1 + (w_1)^2 + (w_2)^2) \\ &= -\rho + \mu(1 + \cos^2\theta(w_r)^2 - \frac{2}{r}cos\theta sin\theta w_r w_\theta + \frac{1}{r^2}\ \sin^2\theta(w_\theta)^2 + \ \sin^2\theta(w_r)^2 \\ &+ \frac{2}{r}cos\theta sin\theta w_r w_\theta + \frac{1}{r^2}\cos^2\theta(w_\theta)^2) \end{aligned}$$

$$-\rho + \mu(1 + (w_1)^2 + (w_2)^2) = -\rho + \mu(1 + (w_r)^2 + \frac{1}{r^2}(w_{,\theta})^2)$$

using the value of $w_1,\ w_{2\,,}\ (w_1)^2\ and\ (w_2)^2$ in equation (10) we obtain

$$\tau_{xx} = \tau_{yy} = \mu - \rho$$

$$\tau_{xy}\ = \tau_{yx\,,} = 0$$

$$\tau_{xz} = \tau_{zx} = \mu(cos\theta w_r - \frac{sin\,\theta}{r} w_\theta)$$
$$\tau_{yz} = \tau_{yz} = \mu(sin\theta w_r + \frac{cos\theta}{r} w_\theta)$$

$$\tau_{zz} = \mu(1 + \left((w_r)^2 + \frac{1}{r^2}(w_\theta)^2\right)) - \rho$$

The coordinate transformation between cylindrical polar coordinate and Cartesian coordinate is given by

$$\tau^* = Q\tau Q^T \tag{15}$$

where Q is an orthogonal transformation tensor and Q^T is the transpose. Then (15) becomes

$$\tau^* = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tau^* = \begin{pmatrix} cos\theta\tau_{xx} + sin\theta\tau_{yx} & cos\theta\tau_{xy} + sin\theta\tau_{yy} & cos\theta\tau_{xz} + sin\theta\tau_{yx} \\ -sin\theta\tau_{xx} + cos\theta\tau_{yx} & -sin\theta\tau_{xy} + cos\theta\tau_{yy} & -sin\theta\tau_{xz} + cos\theta\tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{16}$$

using (10b) in equation (16), we obtain

$$\tau^* = \begin{pmatrix} cos^2\theta\tau_{xx} + sin^2\theta\tau_{yy} & -cos\theta sin\theta\tau_{xy} + cos\theta sin\theta\tau_{yy} & cos\theta\tau_{xz} + sin\theta\tau_{yz} \\ -cos\theta sin\theta\tau_{xx} + cos\theta sin\theta\tau_{yy} & sin^2\theta\tau_{xx} + cos^2\theta\tau_{yy} & -sin\theta\tau_{xz} + cos\theta\tau_{yx} \\ cos\theta\tau_{zx} + sin\theta\tau_{zy} & -sin\theta\tau_{zx} + cos\theta\tau_{zy} & \tau_{zz} \end{pmatrix} \tag{17}$$

In components form of the polar cylindrical material

$$\tau^* = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix} \tag{18}$$

Comparing (17) and (18), we have

$$\tau_{rr} = cos^2\theta\tau_{xx} + sin^2\theta\tau_{yy} \tag{19a}$$

$$\tau_{r\theta} = \tau_{\theta r} = -cos\theta sin\theta\tau_{xy} + cos\theta sin\theta\tau_{yy} \tag{19b}$$

$$\tau_{\theta\theta} = sin^2\theta\tau_{xx} + cos^2\theta\tau_{yy} \tag{19c}$$

$$\tau_{rz} = \tau_{zr} = cos\theta\tau_{xz} + sin\theta\tau_{yz} \tag{19d}$$

$$\tau_{z\theta} = -sin\theta\tau_{zx} + cos\theta\tau_{zy} \tag{19e}$$

$$\tau_{zz} = \tau_{zz} \tag{19f}$$

Using equation (10) in (19), we have

$$\tau_{rr} = \mu - \rho \tag{20a}$$

$$\tau_{\theta\theta} = \mu - \rho \tag{20b}$$

$$\tau_{r\theta} = \tau_{\theta r} = o \tag{20c}$$

$$\tau_{rz} = \tau_{zr} = \mu w_r \tag{20d}$$

$$\tau_{\theta z} = \tau_{z\theta} = \frac{\mu}{r} w_\theta \tag{20e}$$

$$\tau_{zz} = \mu\left((w_r)^2 + \frac{1}{r^2}(w_\theta)^2\right) - \rho \tag{20f}$$

4 Equation of Equilibrium

The equilibrium equation is given by

$$\text{Div}\overline{\tau^*}=0 \tag{21}$$

In component form, we have

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) + B_r = 0 \tag{22a}$$

$$\frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2}{r}\tau_{r\theta} + B_\theta = 0 \tag{22b}$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r}\tau_{zr} + B_z = 0 \tag{22c}$$

where $B_r, B_\theta \; and \; B_z$ are components of the body forces in r, θ and z vectors respectively. The non-zero component of equilibrium equation is the axial component given by

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r}\tau_{zr} + B_z = 0 \tag{23}$$

In the absence of body force (23) reduces to

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r}\tau_{zr} = 0$$

where $\frac{\partial \tau_{zr}}{\partial r} = \mu w_{rr}, \frac{\partial \tau_{z\theta}}{\partial \theta} = \frac{\mu}{r} w_{\theta\theta}, \frac{\partial \tau_{zz}}{\partial z} = 0$

we obtain the equilibrium equation as

$$\mu w_{rr} + \frac{\mu}{r} w_r + \frac{\mu}{r^2} w_{\theta\theta} = 0$$

$$\mu(w_{rr} + \frac{1}{r} w_r + \frac{1}{r^2} w_{\theta\theta}) = 0 \tag{24}$$

where $\mu \neq 0$
then equation (14) becomes

$$w_{rr} + \frac{1}{r} w_r + \frac{1}{r^2} w_{\theta\theta} = 0$$

$$r^2 w_{rr} + r w_r + w_{\theta\theta} = 0$$

$$r^2 w_{rr} + w_{\theta\theta} = -r w_r \tag{25}$$

The standard monge's equation is:

$$R^{*}r^{*} + Ss + Tt^{*} = V \qquad (26)$$

Comparing (27) and (28) we have

$$R^{*} = r^{2}, T = 1, S = 0, V = -rw_{r} \qquad (27)$$

Any equation that satisfies (26) must satisfy

$$R^{*}dpdt + Tdqdr - Vdrdt = 0 \qquad (28a)$$

$$R^{*}(dt)^{2} - Sdrdt + T(dt)^{2} = 0 \qquad (28b)$$

Then (28a) and (28b) becomes

$$r^{2}dpd\theta + dqdR + rw_{r}drd\theta = 0 \qquad (29a)$$

$$r^{2}(d\theta)^{2} + (dr)^{2} = 0 \qquad (29b)$$

From (29b) , we have

$$dr = \pm\sqrt{-r^{2}(d\theta)^{2}} = \pm ird\theta \qquad (30)$$

Substituting (30) in (29a), we obtain

$$r^{2}dpd\theta + irdqd\theta + rw_{r}drd\theta = 0 \qquad (31a)$$

$$r^{2}dpd\theta - irdqd\theta + rw_{r}drd\theta = 0 \qquad (31b)$$

Adding (31a) and (31b) gives

$$2r^{2}dpd\theta + 2\,rw_{r}drd\theta = 0$$

Factoring $d\theta, where\ d\theta \neq 0\ then\ we\ have$

$$r^{2}dp + rw_{r}dr = 0$$

Let $p = w_{r}$ where $w = w(r,\theta)$

$$r^{2}dp = -rpdr$$

$$\int \frac{dp}{p} = -\int \frac{dr}{r}$$

$$Inp = -Inr + InH(\theta)$$

$where\ H(\theta) > 0$ and $\theta \geq 0$

$$p = \frac{H(\theta)}{r}$$

$$w_{r} = \frac{H(\theta)}{r}$$

$$\frac{\partial w}{\partial r} = \frac{H(\theta)}{r}$$

$$w = H(\theta)Inr + F(\theta) \qquad (32)$$

where $r > 0$

5 Use of Boundary Conditions

$$w(a,\theta) = 0, w(b,\theta) = kbsin\theta \tag{33}$$

where k is a constant which is the magnitude of shear

Using equation (33) in (32) gives (34) and (35)

$$0 = H(\theta)In a + F(\theta) \tag{34}$$

$$kbsin\theta = H(\theta)In b + F(\theta) \tag{35}$$

Solving for $F(\theta)$ and $H(\theta)$, we obtain

$$H(\theta) = \frac{kbsin\,\theta}{In(\alpha)} \text{ and } F(\theta) = \frac{-kbsin\theta}{In\alpha} In a \tag{36}$$

where $\alpha = \frac{b}{a}$ and $\alpha > 0$ $since\ b > a$

then (32) gives

$$w(r,\theta) = \frac{\lambda sin\theta}{In\alpha}\, In\frac{r}{a} \tag{37}$$

Equation (37) satisfies the above boundary conditions given in equation (33).

$$w_r = \frac{\lambda sin\theta}{rIn\alpha}$$

$$w_\Theta = \frac{\lambda cos\theta}{In\alpha}\, In\frac{r}{a}$$

where $kb = \lambda$
The stress components become

$$\tau_{rr} = \tau_{\theta\theta} = \mu - \rho$$

$$\tau_{r\theta} = \tau_{\theta r} = o$$

$$\tau_{rz} = \tau_{zr} = \mu\frac{\lambda sin\theta}{In\alpha}\frac{1}{r}$$

$$\tau_{\theta z} = \tau_{z\theta} = \frac{\mu}{r}\frac{\lambda cos\theta}{In\alpha}\, In\frac{r}{a}$$

$$\tau_{zz} = \mu\left((\frac{\lambda sin\theta}{In\alpha}\frac{1}{r})^2 + \frac{1}{r^2}(\frac{\lambda cos\theta}{In\alpha}\, In\frac{r}{a})^2\right) - \rho$$

6 Conclusion

This present work establish an exact analytical solution for the angular displacement and stresses for the anti-plane shear deformation of an incompressible Hollow cylinder made of Neo-Hookean material.

Equation (37) gives the displacement. Finally, we obtain the components of the stress at any cross section of the cylinder made of Neo-Hookean material.

Competing Interests

Author has declared that no competing interests exist.

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